Tutorial 2.

Preliminary:

① Nominal Interest Rate VS. Effective Interest Rate:

$1+i = \left[1+\frac{i^{(m)}}{m}\right]^m \Rightarrow i^{(m)} = m\left[(1+i)^{\frac{1}{m}}-1\right]$, $\lim_{m\to\infty} i^{(m)} = \ln(1+i)$.

② Effective Annual Rate of Discount:

$d = \frac{A(1)-A(0)}{A(1)} \Rightarrow A(0) = A(1)(1-d) \Rightarrow A(n) = A(0)(1-d)^{-n}$.

$A(0) = A(1)v \Rightarrow 1-d = v = \frac{1}{1+i} \Rightarrow d = \frac{i}{1+i}$, or $i = \frac{d}{1-d}$.

③ The Force of Interest: (relative instantaneous rate of growth per unit amount invested)

$\delta_t = \frac{A'(t)}{A(t)}$, i) simple interest rate: $A(t) = A(0)[1+it]$, $A'(t) = A(0)\cdot i$, then $\delta_t = \frac{A'(t)}{A(t)} = \frac{i}{1+it}$ at time t

ii) compound rate: $A(t) = A(0)(1+i)^t$, $A'(t) = A(0)(1+i)^t \cdot \ln(1+i)$, then $\delta_t = \ln(1+i)$

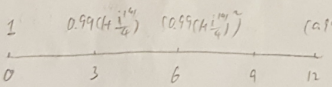
$1+i = e^{\delta_t}$

④ Inflation:

The real rate with interest rate i and inflation rate r.

$i_{real} = \frac{i-r}{1+r}$.

1.4.5.



$i^{(4)} = 3.25\%$, $A(0) = 1$,

$A(12) = (0.99(1+\frac{i^{(4)}}{4}))^4 = 0.992198$.

effective rate $= \frac{A(12)-A(0)}{A(0)} = -0.0078$.

1.5.5.

$A(n) = A(0)(1-d)^{-n}$, $A_B(0) = 100$, $A_R(0) = 50$.

$A_B(11) - A_B(10) = A_R(17) - A_R(16) = X$.

$100\left[(1-d)^{-11} - (1-d)^{-10}\right] = 50\left[(1-d)^{-17} - (1-d)^{-16}\right]$

$\Rightarrow 100(1-d)^{-10} = 50(1-d)^{-16} \Rightarrow d = 0.1091$.

$X = 38.9$.

1.6.6

Bruce: $A(t) = A(0)(1+i)^t = A(0)\left[\left(1+\frac{i^{(m)}}{m}\right)^m\right]^t$.

$200 = 100\left[\left(1+\frac{i}{2}\right)^2\right]^{7.25} = 100\left(1+\frac{i}{2}\right)^{14.5} = 200$.

$i = 0.0979$.

Peter: $A(t) = \cancel{A(0)e^{\delta t}} \Rightarrow$

$A(0)(1+i)^t = A(0)e^{\delta t}$ $200 = 100e^{7.25\delta} \Rightarrow \delta = 0.0956$. $\Rightarrow i - \delta = 0.23\%$.

$(i^{(m)} > i^{(\infty)})$.

1.7.2.

$i_{real} = \frac{i-r}{1+r}$, $i = 0.12 \times (1-0.25)$ (after-tax), $r = 0.1$. $i_{real} = \frac{0.12 \times 0.75 - 0.1}{1+0.1} = -0.0309$.